

# Linear Equations

NAME Key

Decide whether each of the following equations is an identity or a conditional equation.

1. $2y - y^2 = y(y - 2)$ $2y - y^2 = y^2 - 2$ Conditional	2. $m^2 - 4 = (m + 2)(m - 2)$ Identity
3. $2x + 5 = 2(x + 5)$ Contradiction	4. $3x + 8 - x = 3(x + 8) - x$ Contradiction
5. $2(x - 1) = x - 1 + x - 1$ Identity	6. $4p + 16 = 5(p + 4) - (p + 4)$ Identity

Decide which of the following pairs of equations are equivalent.

7. $4x - 1 = 10$ $x = 11/4$ $12x - 3 = 30$ $x = 31/12$ Equivalent	8. $5 = 8 - 2x$ $2x = 3$ Equivalent	9. $\frac{y+2}{y+3} = \frac{4}{y+3}$ $y + 2 = 4$ Equivalent	10. $\frac{2x+5}{9} = \frac{4x}{9}$ $5 = 2x$ Equivalent
11. $\frac{x}{x-2} = \frac{2}{x-2}$ $x = 2$ Not Equivalent	12. $\frac{x+3}{x+1} = \frac{2}{x+1}$ $x = -1$ Not Equivalent	13. $x = 4$ $x^2 = 16$ Not Equivalent	14. $z^2 = 9$ $z = 3$ Not Equivalent

Solve each of the following equations. Do not use a calculator. When solving problems with fractions and decimals eliminate the fractions and decimals in the equation.

15. $(.3x - .7 = .3 + 2x) / 10$ $3x - 7 = 3 + 20x$ $-17x = 10$ $x = -\frac{10}{17}$	16. $(.04x - 2.01 = 3.18 + 4.72) / 100$ $4x - 201 = 318 + 472$ $4x - 201 = 790$ $4x = 991$ $x = 991/4$
17. $(\frac{3}{4}x - 5 + \frac{2}{3} = \frac{5}{3} - x) / 12$ $9x - 60 + 8 = 20 - 12x$ $9x - 52 = 20 - 12x$ $21x = 72$ $x = \frac{72}{21} = \frac{24}{7}$	18. $(-\frac{1}{2} + \frac{1}{4}y + 2 = \frac{3}{4}y) / 4$ $-2 + 4 + 8 = 3y$ $6 = 2y$ $3 = y$

19.  $3(2a+1) - 4 = 3a - 7[2(3a-1) + 6]$   
 $6a+3-4 = 3a-7(6a-2+6)$   
 $6a-1 = 3a-7(6a+4)$   
 $6a-1 = 3a-42a-28$   
 $6a-1 = -39a-28$   
 $45a = -27$   
 $a = \frac{-27}{45} = -\frac{3}{5}$  ✓

20.  $5k + 2[-4(k+3) - k] = 4 - (2k+3)$   
 $5k + 2(-4k-12-k) = 4-2k-3$   
 $5k - 8k - 24 - 2k = 4-2k-3$   
 $-5k - 24 = 1-2k$   
 $-3k = 25$   
 $k = \frac{-25}{3}$  ✓

21.  $\left(\frac{8}{3}(6x+7) - 8x = \frac{1}{2}(4-x) + 11\right) 6$   
 $16(6x+7) - 48x = 3(4-x) + 66$   
 $96x + 112 - 48x = 12 - 3x + 66$   
 $48x + 112 = 78 - 3x$   
 $51x = -34$   
 $x = \frac{-34}{51} = -\frac{2}{3}$  ✓

22.  $\left(7 - \frac{1}{2}(5y-3) = 10 - \frac{4}{5}(3-4y)\right) 10$   
 $70 - 5(5y-3) = 100 - 8(3-4y)$   
 $70 - 25y + 15 = 100 - 24 + 32y$   
 $85 - 25y = 76 + 32y$   
 $-57y = -9$   
 $y = \frac{-9}{-57} = \frac{3}{19}$  ✓

23.  
 $2(5a - 3a^2) + (3a - 1)(2a + 3) = 8a + 12$   
 $10a - 6a^2 + 6a^2 + 7a - 3 = 8a + 12$   
 $17a - 3 = 8a + 12$   
 $9a = 15$   
 $a = \frac{15}{9}$   
 $a = \frac{5}{3}$  ✓

24.  $(-.4(5k-.4) = 1.3 - .6(.3 - .2k)) 100$   
 $-4(5k-.4) = 130 - 6(.3 - .2k)$   
 $-20k + 16 = 130 - 18 + 12k$   
 $-20k + 16 = 112 + 12k$   
 $-32k = 96$   
 $k = \frac{96}{-32}$   
 $k = -3$  ✓

25.  
 $4x - 7 - (x-1)(x+1) = -3 - (x-2)(x-3)$   
 $4x - 7 - (x^2-1) = -3 - (x^2-5x+6)$   
 $4x - 7 - x^2 + 1 = -3 - x^2 + 5x - 6$   
 $4x - 6 = 5x - 9$   
 $-x = -3$   
 $x = 3$  ✓

26.  $(.2(1-5x) = -.8(x-.1)) 100$   
 $2(1-50x) = -8(10x-1)$   
 $2 - 100x = -80x + 8$   
 $-20x = 6$   
 $x = \frac{-6}{20}$   
 $x = -\frac{3}{10}$  ✓

27.  $\frac{3x-2}{7} = \frac{x+2}{5}$   
 $15x - 10 = 7x + 14$   
 $8x = 24$   
 $x = 3$

28.  $\frac{2p+5}{5} = \frac{p+2}{3}$   
 $6p + 15 = 5p + 10$   
 $p = -5$  ✓

<p>29. <math>\left(\frac{9}{x+7} - \frac{3}{2x+1} = \frac{7}{x+7}\right)(x+7)(2x+1)</math>  <math>9(2x+1) - 3(x+7) = 7(2x+1)</math>  <math>18x+9 - 3x-21 = 14x+7</math>  <math>15x-12 = 14x+7</math>  <math>x = 19</math> ✓</p>	<p>30. <math>\frac{6x+7}{4x-1} = \frac{3x+8}{2x-4}</math>  <math>(6x+7)(2x-4) = (4x-1)(3x+8)</math>  <math>12x^2 - 10x - 28 = 12x^2 + 29x - 8</math>  <math>-39x = 20</math>  <math>x = -\frac{20}{39}</math> ✓</p>
<p>31. <math>\left(\frac{2x}{x+3} = \frac{-6}{x+3} - 2\right)x+3</math> <math>x \neq -3</math>  <math>2x = -6 - 2(x+3)</math>  <math>2x = -6 - 2x - 6</math>  <math>4x = -12</math>  <math>x = -3</math> <math>\emptyset</math> ✓</p>	<p>32. <math>\left(\frac{y+3}{y-6} + \frac{y-2}{y+2} = \frac{2y^2+9}{y^2-4y-12}\right)(y-6)(y+2)</math>  <math>(y+3)(y+2) + (y-2)(y-6) = 2y^2+9</math>  <math>y^2+5y+6 + y^2-8y+12 = 2y^2+9</math>  <math>-3y+18 = 9</math>  <math>-3y = -9</math>  <math>y = 3</math> ✓</p>
<p>33. <math>\left(\frac{3x}{7x+14} + \frac{5}{7} = \frac{1}{x+2} - \frac{13}{7x+14}\right)7(x+2)</math> <math>x \neq -2</math>  <math>3x + 5(x+2) = 7 - 13</math>  <math>8x+10 = -6</math>  <math>8x = -16</math>  <math>x = -2</math> <math>\emptyset</math> ✓</p>	<p>34. <math>\left(\frac{x}{x^2-1} - \frac{x+3}{x^2-x} = \frac{-3}{x^2+x}\right)(x-1)(x+1)(x)</math>  <math>x^2 - (x+3)(x+1) = -3(x-1)</math>  <math>x^2 - (x^2+4x+3) = -3x+3</math>  <math>-4x-3 = -3x+3</math>  <math>-x = 6</math>  <math>x = -6</math> ✓</p>

## LITERAL EQUATIONS

Solve for x.

<p>35. <math>2(x-a) + b = 3x + a</math>  <math>2x - 2a + b = 3x + a</math>  <math>2x - 3x = 3a - b</math>  <math>-x = 3a - b</math>  <math>x = b - 3a</math></p>	<p>36. <math>5x - (2a + c) = a(x + 1)</math>  <math>5x - 2a - c = ax + a</math>  <math>5x - ax = 3a + c</math>  <math>x(5-a) = 3a + c</math>  <math>x = \frac{(3a+c)}{(5-a)}</math> <math>a \neq 5</math></p>
<p>37. <math>ax + b = 3(x - a)</math>  <math>ax + b = 3x - 3a</math>  <math>ax - 3x = -3a - b</math>  <math>x(a-3) = -3a - b</math>  <math>x = \frac{(-3a-b)}{(a-3)}</math> <math>a \neq 3</math></p>	<p>38. <math>4a - ax = 3b + bx</math>  <math>4a - 3b = ax + bx</math>  <math>4a - 3b = x(a+b)</math>  <math>x = \frac{4a-3b}{a+b}</math> <math>a \neq -b</math></p>
<p>39. <math>\frac{4x}{2a+1} = ax - 1</math> <math>4x = 2a^2x - 2a + ax - 1</math>  <math>2a+1 = 2a^2x + ax - 4x</math>  <math>2a+1 = x(2a^2+a-4)</math>  <math>x = \frac{2a+1}{2a^2+a-4}</math></p>	<p>40. <math>\frac{a}{3x+2} + b = 2a</math>  <math>\frac{a}{3x+2} = 2a - b</math>  <math>x = \frac{-3a+2b}{6a-3b}</math>  <math>a = \frac{(2a-b)(3x+2)}{6a-3b}</math>  <math>a = 6ax + 4a - 3bx - 2b</math> <math>b \neq 2a</math></p>

41.  $a^2(2x - 3) = 4x$

$$\begin{aligned} 2a^2x - 3a^2 &= 4x \\ 2a^2x - 4x &= 3a^2 \\ x(2a^2 - 4) &= 3a^2 \\ x &= \frac{3a^2}{2a^2 - 4} \end{aligned}$$

$a \neq \pm 2$

42.  $a(x + a) = b(x + b)$

$$\begin{aligned} ax + a^2 &= bx + b^2 \\ ax - bx &= b^2 - a^2 \\ x(a - b) &= b^2 - a^2 \\ x &= \frac{b^2 - a^2}{a - b} \end{aligned}$$

Solve for the indicated variable.

$$x = \frac{\frac{-1}{(b-a)}(b+a)}{(a-b)} = \frac{-1}{-(b+a)} = \frac{1}{b+a}$$

43.  $kr - p = br + c$  for  $r$

$$\begin{aligned} kr - br &= p + c \\ r(k - b) &= p + c \\ r &= \frac{p + c}{k - b} \end{aligned}$$

$k \neq b$

44.  $A = \frac{1}{2}(B + b)h$  for  $h$

$$\begin{aligned} 2A &= (B + b)h \\ h &= \frac{2A}{B + b} \end{aligned}$$

$B \neq -b$

45.  $C = \frac{5}{9}(F + 32)$  for  $F$

$$\begin{aligned} \frac{9}{5}C &= F + 32 \\ F &= \frac{9}{5}C - 32 \end{aligned}$$

46.  $A = P(1 + \frac{i}{m})$  for  $m$

$$\begin{aligned} \frac{A}{P} &= 1 + \frac{i}{m} \\ \frac{A}{P} - 1 &= \frac{i}{m} \\ m &= \frac{Pi}{A - P} \end{aligned}$$

$A \neq P$

47.  $m = \frac{Ft}{v_1 - v_2}$  for  $v_2$

$$\begin{aligned} v_1 - v_2 &= \frac{Ft}{m} \\ -v_2 &= \frac{Ft}{m} - v_1 \end{aligned}$$

$$v_2 = \frac{v_1 m - Ft}{m}$$

$m \neq 0$

48.  $\frac{1}{a} = \frac{1}{b} + \frac{1}{c}$  for  $c$

$$\begin{aligned} bc &= ac + ab \\ bc - ac &= ab \\ c(b - a) &= ab \end{aligned}$$

$$c = \frac{ab}{b - a}$$

$a \neq b$

49.  $\frac{x - a}{y - b} = 2$ ; for  $y$

$$\begin{aligned} x - a &= 2y - 2b \\ 2y &= x - a + 2b \\ y &= \frac{x - a + 2b}{2} \end{aligned}$$

50.  $\frac{2b}{c - 1} = b - d$ ; for  $c$

$$\begin{aligned} \frac{2b}{b - d} &= c - 1 \\ \frac{2b}{b - d} + 1 &= c \end{aligned}$$

$$c = \frac{2b + b - d}{b - d} = \frac{3b - d}{b - d}$$

$b \neq d$

51.  $y = \frac{ax + b}{cx + d}$ ; for  $x$

$$\begin{aligned} ycx + dy &= ax + b \\ ycx - ax &= b - dy \\ x(yc - a) &= b - dy \\ x &= \frac{b - dy}{yc - a} \end{aligned}$$

$yc \neq a$

52.  $S = k(1 + \frac{r}{n})$ ; for  $r$

$$\begin{aligned} \frac{S}{k} &= 1 + \frac{r}{n} \\ \frac{S - k}{k} &= \frac{r}{n} \end{aligned}$$

$$r = \frac{S(n - k)}{k}$$

$k \neq 0$